Dicovers in Ditopological Texture space

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Abstract

The aim of this paper deals with the new concept namely β- para compactness in ditopological texture spaces.Also we develope some theorems using paracompactness and β-open sets. Many effective characterizations and properties of this newly developed concept are obtained.

Keywords :Texture spaces,Ditopology, Ditopological Texture spaces,β-paracompactness, β-locally finite,β-locally co-finite

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1   1.Introduction

In 1998 L.M.Brown introduced on attractive concept namely Textures in ditopological setting for the study of fuzzy sets in 1998. A systematic development of this texture in ditopology has been extensively made by many researchers[3,4,5,7].

The present study aims at discussing the effect of β-paracompactness in Ditopological Texture spaces. In Ditopological Texture Spaces: Let S be a set, a texturing T of S is a subset of P(S). If

(1) (T,$\subseteq$) is a complete lattice containing S and ϕ, and the meet and join operations in (T,$\subseteq$) are related with the intersection and union operations in (P(S),$\subseteq$)by the equalities

$\wedge_{i \in I} A_i = \cap_{i \in I} A_i, A_i \in T, i \in I$, for all index sets I, while

$\vee_{i \in I} A_i = \cup_{i \in I} A_i, A_i \in T, i \in I$, for all finite index sets I.

(2) T is completely distributive.

(3) T separates the points of S. That is, given $s_1 \neq s_2$ in S we have $A \in T with s_1 \in A, s_2 \notin A, or A \in T$ with $s_2 \in A, s_1 \notin A$.

If S is textured by T we call (S,T) a texture space or simply a texture.

For a texture (S; T), most properties are conveniently defined in terms of the p-sets $P_s = \cap \{A \in T \backslash s \in A\}$ and the q-sets, $Q_s = \vee \{A \in T / s \notin A\}$: The following are some basic examples of textures.

The notion of stability for bitopological spaces was introduced by Ralph Kopperman The analogous notion, and its dual, were given for ditopologies in [5], and studied in greater detail in [11]. We now wish to generalize these concepts for β-open and β-closed sets.

Examples 1.1. Some examples of texture spaces,

(1) If X is a set and P(X) the powerset of X, then (X; P(X)) is the discrete texture on X. For $x \in X, P_x = \{x\}$ and $Q_x = X \backslash \{x\}$.

(2) Setting I = [0; 1], T= $\{[0; r); [0; r]/r \in I\}$ gives the unit interval texture (I; T). For $r \in I, P_r = [0; r]$ and $Q_r = [0; r)$.

(3) The texture (L;T) is defined by L = (0; 1], T = $\{(0; r]/r \in I\}$. For $r \in L, P_r = (0; r] = Q_r$.

(4) T=$\{\phi, \{a, b\}, \{b\}, \{b, c\}, S\}$ is a simple textureing of S = $\{a, b, c\}$ clearly $P_a = \{a, b\}, P_b = \{b\} and P_c = \{b, c\}$.

Since a texturing T need not be closed under the operation of taking the set complement, the notion of topology is replaced by that of dichotomous topology or ditopology, namely a pair (τ, κ) of subsets of T, where the set of open sets τ satisfies

1. S,$\phi \in \tau$,

2. $G_1; G_2 \in \tau$ then $G_1 \cap G_2 \in \tau$ and

3. $G_i \in \tau$, i$\in I$ then $\vee_i G_i \in \tau$,

and the set of closed sets κ satisfies

1. S, $\phi \in \kappa$

2. $K_1; K_2 \in \kappa$ then $K_1 \cup K_2 \in \kappa$ and

3. $K_i \in \kappa, i \in$ I then $\cap K_i \in \kappa$. Hence a ditopology is essentially a 'topology" for which there is no a priori relation between the open and closed sets.

For A $\in$ T we define the closure [A] or cl(A) and the interior]A[or int(A) under (τ, κ) by the equalities [A] $= \cap\{K \in \kappa / A \subset K\}$ and]A[$= \vee\{G \in \tau / G \subset A\}$:

Definition 1.2. For a ditopological texture space (S; T; τ, κ):

1. A $\in$ T is called pre-open (resp. semi-open,β-open) if A $\subseteq$ intclA (resp. A $\subseteq$ clintA;A $\subseteq$ clintclA). B $\in$ T is called pre-closed (resp. semi-closed,β-closed) if clintB $\subseteq$ B (resp. intclB $\subseteq$ B; intclintB $\subseteq$ B)

We denote by PO(S; T;τ, κ)($\beta O(S; T; \tau, \kappa)$), more simply by PO(S) (βO(S)) , the set of pre-open sets (β-open sets) in S. Likewise, PC(S; T;τ, κ) ($\beta C(S; T; \tau, \kappa)$), PC(S) ($\beta$ C(S)) will denote the set of pre-closed (β-closed sets) sets.

As in [3]we will also consider the sets $Q_s \in T, s \in S$, defined by

$$Q_s = \vee\{P_t | s \notin P_t\}$$

. By [1.1] examples, we have $Q_x = X/\{x\}, Q_r = (0, r] = P_r and Q_t = [0, t)$ respectively. The second example shows clearly that we can have $s \in Q_s$, and indeed even $Q_s = S$. Also, in general, the sets Q_s do not have any clear relation with either the set theoretic complement or the complementation on T. They are, however, closely connected with the notion of the core of the sets in S.

Definition.1.3 For $A \in T$ the core of A is the set $core(A) = \cap\{\cup\{A_i | i \in I\} | A = \vee\{A_i | A_i \in T, i \in I\}\}$.

Clearly core(A) $\subseteq$ A, and in general we can have core(A) $\notin T$. We will generally denote core(A) by A^b.

Lemma 1.4.[1] (1) $s \notin A \Rightarrow A \subseteq Q_s \Rightarrow s \notin A^b$ for all $s \in S, A \in T$.

(2) $A^b = \{s | A \nsubseteq Q_s\} for all A \in T$.

(3) For $A_i \in T, i \in I$, we have $(\vee_{i \in I} A_i)^b = \cup_{i \in I} A_i^b$.

(4) A is the smallest element of T containing A^b for all $A \in T$.

(5) For A, B$\in T, if A \nsubseteq$B, then there exists $s \in S with A \nsubseteq Q_s$ and $P_s \nsubseteq B$.

(6) A = $\cap \{Q_s | s \notin A\}$ for all $A \in T$.

2   2. Dicovers and β-locally finite

Definition 2.1 *A subset* $\mathcal{C}$ *of* $T \times T$ *is called a difamily on (S,T). Let* $\mathcal{C}$=$\{(G_\beta, F_\beta)/\beta \in A\}$ *be a family on (S,T).Then T is called a dicover of (S,T) if for all partitions* A_1, A_2 *of A, we have*

$$\cap_{\beta \in A_1} F_\beta \subseteq \vee_{\beta \in A_2} G_\beta$$

Definition 2.2 *Let* (τ, κ) *be a ditopology on (S,T). Then a difamily* $\mathcal{C}$ *on (S, T,τ,κ). is called β-open(co-β-open) if dom($\mathcal{C}$)$\subseteq \beta O(S)$ (ran($\mathcal{C}$)$\subseteq \beta O(S)$).*

Definition 2.3 *Let* (τ, κ) *be a ditopology on (S,T). Then a difamily* $\mathcal{C}$ *on (S, T,τ,κ). is called β-closed(co-β closed) if dom($\mathcal{C}$)$\subseteq \beta C(S)$. (ran($\mathcal{C}$)$\subseteq \beta C(S)$).*

Lemma 2.4 [3] *Let (S, T) be a texture. Then P=$\{(P_s, Q_s)|s \in S^b\}$ is a dicover of S.*

Corollary 2.5 [3] *Given $A \in T, A \neq \emptyset$, there exists $s \in S^b$ with $P_s \subseteq A$.*

Definition 2.6 *Let (S, T) be a texture, C and $\mathcal{C}'$ difamilies in (S, T). Then $\mathcal{C}$ is said to be a refinement of $\mathcal{C}'$, written $\mathcal{C} < \mathcal{C}'$. If given A $\mathcal{C}$ B we have*
A' $\mathcal{C}'$ B' with $A \subseteq A'$ and $B' \subseteq B$.If $\mathcal{C}$ is a dicover and $\mathcal{C} < \mathcal{C}'$, then clearly $\mathcal{C}'$ is a dicover.

Remark 2.7 *Given dicovers $\mathcal{C}$ and $\mathcal{D}$ then $\mathcal{C} \wedge \mathcal{D}$ =$\{(A \cap C, B \cup D)|A \mathcal{C} B, C \mathcal{D} D\}$ is also a dicover. It is the meet of C and D with respect to the refinement relation.*

Definition 2.8 *Let* $\mathcal{C} = \{G_i, F_i)|i \in I\}$ *be a difamily indexed over I. Then* $\mathcal{C}$ *is said to be*

(i)Finite (co-finite) if dom($\mathcal{C}$) (resp., ran($\mathcal{C}$)) is finite.

(ii) β-Locally finite if for all $s \in S$ there exists $K_s \in \beta C(S)$ with $P_s \not\subseteq K_s$ so that the set
$\{i|G_i \not\subseteq K_s\}$ *is finite.*

(iii) β-Locally co-finite if for all $s \in S$ with $Q_s \neq S$ there exists $H_s \in \beta O(S)$ with $H_s \not\subseteq Q_s$
so that the set $\{i|H_s \not\subseteq F_i\}$ is finite.

(iv) Point finite if for each $s \in S$ the set $\{i|P_s \subseteq G_i\}$ is finite.

(v) Point co-finite if for each $s \in S$ with $Q_s \neq S$ the set $\{i|F_i \subseteq Q_s\}$ is finite.

Lemma 2.9 *Let (S,T,τ,κ) be a ditopological texture space and $\mathcal{C}$ be a difamily then, the*
following are equivalent:

1. $\mathcal{C}$=$\{(G_i, F_i)|i \in I\}$ *is β locally finite.*

2. *There exists a family $\mathcal{B} = \{B_j|j \in J\} \subseteq T/\{\emptyset\}$ with the property that for $A \in T$ with*
   $A \neq \emptyset$, *we have $j \in J$ with $B_j \subseteq A$, and for each $j \in J$ there is $K_j \in \beta C(S)$ so that*
   $B_j \not\subseteq K_j$ *and the set $\{i|G_i \not\subseteq K_j\}$ is finite.*

Proof. Straightforward.

Lemma 2.10 *Let (S,T,τ,κ) be a ditopological texture space and $\mathcal{C}$ be a difamily then, the*
following are equivalent:

(a) $\mathcal{C}$=$\{(G_i, F_i)|i \in I\}$ is β-locally co-finite.

(b) There exists a family $\mathcal{B} = \{B_j|j \in J\} \subseteq T/\{S\}$ with the property that for $A \in T$ with
$A \neq S$, *we have $j \in J$ with $A \subseteq B_j$, and for each $j \in J$ there is $H_j \in \beta O(S)$ so that $H_j \not\subseteq B_j$*
and the set $\{i|H_i \not\subseteq F_j\}$ is finite.

Theorem 2.11 *The difamily $\mathcal{C}$=$\{(G_i, F_i)|i \in I\}$ is β locally finite if for each $s \in S$ with*
$Q_s \neq S$ *we have $K_s \in \beta C(S)$ with $P_s \not\subseteq K_s$, so that the set $\{i|G_i \not\subseteq K_s\}$ is finite.*

Proof. Given $\mathcal{C}=\{(G_i, F_i)|i \in I\}$ is β locally finite ,then by Lemma 5.4.9 there exists a family $\mathcal{B} = \{B_j|j \in J\} \subseteq T/\{\emptyset\}$ with the property that for $A \in T$ with $A \neq \emptyset$, we have $j \in J$ with $B_j \subseteq A$, and for each $j \in J$ there is $K_j \in \beta C(S)$ so that $B_j \nsubseteq K_j$ and the set $\{i|G_i \nsubseteq K_j\}$ is finite. Now take $\mathcal{B} = \{P_s|Q_s \neq S\} = \{P_s|s \in S^b\}$, and for $A \in T$ and A is nonempty, then by corollary 5.4.5 there exists $s \in S^b$ with $P_s \subseteq A$.

Therefore for every $s \in S$, there exists $K_s \in \beta C(S)$ with $P_s \nsubseteq K_s$ such that $\{i|G_i \nsubseteq K_s\}$ is finite.

Theorem 2.12 *Let (S,T,τ,κ) be a ditopological texture space and $\mathcal{C}$ be a β-locally finite dicover and $s \in S$. Then there exists $A \mathcal{C} B$ with $s \in A$ and $s \notin B$.*

Proof. Given $\mathcal{C}=\{(A_i, B_i)|i \in I\}$ be β-locally finite. Take $K\in \beta C(S)$ with $s \notin K$ and $\{i \in I|A_i \nsubseteq K\}$ is finite. Now partition the set I into two sets such that $I_1 = \{i \in I|s \in A_i\}$ and $I_2 = \{i \in I|s \notin A_i$, since $\mathcal{C}$ is a dicover it should satisfy

$$\cap_{i \in I_1}B_i \subseteq \vee_{i \in I_2}A_i$$

now $\vee_{i \in I_2}A_i$ does not have s according to our partition, which implies $s \notin \cap_{i \in I_1}B_i$. Thus we arrived at for all $i \in I_1 s \in A_i$ and $s \notin B_i$.(i.e) $s \in A$ and $s \notin B$.

Theorem 2.13 *Let (S,T,τ,κ) be a ditopological texture space and $\mathcal{C} = \{(\mathcal{A}_\rangle,\mathcal{B}_\rangle)|\rangle \in \mathcal{I}\}$ be a difamily.*

(1) If $\mathcal{C}$ is β-locally finite, then dom($\mathcal{C}$) is β closure preserving.

(2) If $\mathcal{C}$ is β-locally co-finite, then ran($\mathcal{C}$) is β interior preserving.

Proof. (1) Let I' subset of I. We have to prove $\beta cl(\vee_{i \in I'}(A_i)) = \vee_{i \in I'} \beta cl(A_i)$. To prove $\beta cl(\vee_{i \in I'}(A_i)) \subseteq \vee_{i \in I'} \beta cl(A_i)$ suppose this is not true, we get there exists $s \in S$ with $s \in \beta cl(\vee_{i \in I'}(A_i))$ and $s \notin \vee_{i \in I'} \beta cl(A_i)$

$$\beta cl(\vee_{i \in I'}A_i) \nsubseteq Q_s \text{ and } P_s \nsubseteq \vee_{i \in I'}\beta cl(A_i) ------- (*)$$

Since $\mathcal{C}$ is β-locally finite, we have $\{i \in I|A_i \nsubseteq \vee_{i \in I'}cl(A_i)\}$ is finite. Now partition I' into two sets such that

$$I_1 = \{i \in I'|A_i \nsubseteq K\} \text{ and } I_2 = I'/I_1$$

$Now \vee_{i \in I'} A_i = \vee_{i \in I_2} \cup \vee_{i \in I_1} A_i$

$\qquad = \vee_{i \in I_2} \cup \cup_{i \in I_1} A_i$

$\beta cl(\vee_{i \in I'} A_i) \subseteq K \cup \cup_{i \in I_1} A_i$

*using * we can say $\vee_{i \in I'} \beta cl(A_i) \subseteq Q_s$, which is a contradiction.*

Therefore $\beta cl(\vee_{i \in I'}(A_i)) = \vee_{i \in I'} \beta cl(A_i)$. Hence β closure is preserving.

(2) It is the dual of (1).

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