RPS-SEPARATION AXIOMS-II

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ABSTRACT

In this paper, we introduce and study some lower separation axioms using rps-open sets. We discuss their basic properties and their link with existing lower separation axioms.

Keywords and Phrases: pre-closed, rg-closed, semi-pre-closed, pgpr-closed, rps-closed etc.,
MSC 2010: 54A05, 54B05, 54C08

1. Introduction

Shyla Isac Mary and Thangavelu [10] defined the concept of regular pre-semi closed sets in 2010. Askish Kar and Bhattacharyya [3]; Gnanambal [5]; Anitha and Thangavelu[2]; introduced and studied pre-T_i (i=0,1,2), preregular-$T_{1/2}$ and pgpr-$T_{1/2}$, gpr-$T_{1/2}$ spaces respectively. Quit recently, Shyla Isac Mary and Thangavelu [13] introduced and investigated rps-$T_{3/4}$, rps-$T_{1/2}$, rps-$T_{1/3}$ and rps-T_b spaces. The authors [4] further studied pgpr-separation axioms. In this paper, we introduce and study rps-T_i (i=0, 1, 2) spaces.

2. Preliminaries

Given any subset A in a topological space (X,τ), the closure, interior and complement of A are denoted by *cl*(A), *int*(A) and X\A respectively. Let us recall the following definitions, which we shall require later.

A subset A of a topological space (X,τ) is regular open [14] if A=*int*(*cl*(A)), regular closed if A=*cl*(*int*(A)), pre-open [8] if $A\subseteq$*int*(*cl*(A)), pre-closed if *cl*(*int*(A))$\subseteq$A, semi-pre-open [1] if $A\subseteq$*cl*(*int*(*cl*(A))) and semi-pre-closed if *int*(*cl*(*int*(A)))$\subseteq$A. The semi-pre-interior of a subset A of

X is the union of all semi-pre-open sets contained in A and is denoted by *spint*(A). The pre-closure of a subset A of X is the intersection of all pre-closed sets containing A and is denoted by *pcl*(A). The semi-pre-closure of a subset A of X is analogously defined and is denoted by *spcl*(A).

Again a subset B of a topological space (X,τ) is called generalized closed (briefly g-closed) [6] if $cl(B)\subseteq U$ whenever $B\subseteq U$ and U is open in X and regular generalized closed (briefly rg-closed) [8] if $cl(B)\subseteq U$ whenever $B\subseteq U$ and U is regular open in X. The complement of a g-closed set is g-open and that of rg-closed set is rg-open. A subset B of a topological space (X,τ) is called pre-semi-closed [15] if $spcl A\subseteq U$ whenever $A\subseteq U$ and U is g-open.

Definition 2.1: A subset B of a topological space (X,τ) is called generalized pre-regular closed (briefly gpr-closed) [5] (resp. pre-generalized pre-regular-closed (briefly pgpr-closed)[2]) if $pcl(B)\subseteq U$ whenever $B\subseteq U$ and U is regular open(resp. rg-open) in X.

The intersection of all pgpr-closed sets containing A is called the pgpr-closure of A and denoted by *pgpr-cl*(A). The complement of a pgpr-closed set is pgpr-open.

Definition 2.2: A subset B of a topological space (X,τ) is called regular pre-semiclosed (briefly rps-closed) [10]) if $spcl(A)\subseteq U$ whenever $A\subseteq U$ and U is rg-open.

The intersection of all rps-closed sets containing A is called the rps-closure of A and denoted by *rps-cl*(A). The complement of a rps-closed set is rps-open.

Definition 2.3: A topological space (X,τ) is pgpr-T_0 [4] if for any two distinct points x and y of X, there exists a pgpr-open set G such that $x\in G$ and $y\notin G$ or $y\in G$ and $x\notin G$.

Definition 2.4: A topological space (X,τ) is pre-T_1 [3] (resp. pgpr-T_1 [4]) if for any two distinct points x, $y\in X$, there exist pre-open (resp. pgpr-open) sets G and H such that $x\in G$ but $y\notin G$ and $y\in H$ but $x\notin H$.

Definition 2.5: A topological space (X,τ) is pre-T_2 [3] (resp. pgpr-T_2 [4]) if for any two distinct points x, $y\in X$, there exist disjoint pre-open (resp. pgpr-open) sets G and H such that $x\in G$ and $y\in H$.

Theorem 2.6: A subset A of X is rps-open if and only if $F\subseteq spint(A)$ whenever $F\subseteq A$, F is rg-closed. [11]

Definition 2.7[13]: A topological space (X,τ) is called rps-$T_{1/2}$(resp. rps-$T_{1/3}$, rps-T_b and rps-$T_{3/4}$) if every rps-closed (resp. pre-semi-closed, rps-closed and rps-closed) set is semi-pre-closed(resp. rps-closed, semi-closed and pre-closed).

Definition 2.8: A space (X,τ) is called pgpr-$T_{1/2}$[2](resp. gpr-$T_{1/2}$[2] and preregular-$T_{1/2}$ [5]) if every pgpr-closed (resp. gpr-closed and gpr-closed) set is pre-closed (resp. pgpr-closed and pre-closed).

Definition 2.9[11]: A function $f : X\rightarrow Y$ is rps-continuous (resp. rps-irresolute) if $f^{-1}(V)$ is rps-closed for every closed (resp. rps-closed) set V of Y.

Definition 2.10[12]: A function $f : X\rightarrow Y$ is and f is rps-open if $f(G)$ is rps-open in Y for every rps-open set G of X.

Diagram 2.11: closed $\longrightarrow$ pre-closed $\longrightarrow$ pgpr-closed $\longrightarrow$ rps-closed

3. rps-T_0 spaces

In T_0 spaces, two distinct points x, y are separated by means of an open set containing a specific point of x, y and not containing the other. In this section, we introduce rps-T_0 spaces and investigate their basic properties.

Definition 3.1: A topological space (X,τ) is said to be rps-T_0 if for any two distinct points x and y of X there exists a rps-open set G such that $x\in G$ and $y\notin G$ or $y\in G$ and $x\notin G$.

Proposition 3.2: Every topological space is rps-T_0.

Proof: Let (X,τ) be a topological space. By using proposition 3.7 of [4], (X,τ) is pgpr-T_0. Since every pgpr-open set is rps-open, (X,τ) is rps-T_0.

Theorem 3.3: In a topological space (X,τ), the rps-closures of distinct points are distinct.

Proof: Let x and y be two distinct points of a space X. By Proposition 3.2, (X,τ) is rps-T_0. By Definition 3.1, there exists a rps-open set G such that $x\in G$ and $y\notin G$ or $y\in G$ and $x\notin G$. Since G is rps-open, we have $X\backslash G$ is rps-closed. If $x\in G$ and $y\notin G$, then $x\notin X\backslash G$ and $y\in X\backslash G$. Then there is a rps-closed set containing y but not x. It follows that $x\notin \mathit{rps\text{-}cl}(\{y\})$. But $x\in \mathit{rps\text{-}cl}(\{x\})$. Therefore $\mathit{rps\text{-}cl}(\{x\})\neq \mathit{rps\text{-}cl}(\{y\})$. The proof for the case $y\in G$ and $x\notin G$ is similar.

4. rps-T_1 spaces

In this section, we introduce rps-T_1 spaces and investigate their basic properties. In section 3, we have proved that every topological space is rps-T_0. Therefore, it is worth to define rps-T_1 spaces.

Definition 4.1: A topological space (X,τ) is said to be rps-T_1 if for any two distinct points x and y of X, there exist rps-open sets G and H such that $x\in G$ but $y\notin G$ and $y\in H$ but $x\notin H$.

Proposition 4.2: (i) Every pgpr-T_1 space is rps-T_1.

(ii) Every pre-T_1 space is rps-T_1.

Proof: Suppose (X,τ) is pgpr-T_1. Let $x\neq y\in X$. Then by Definition 2.4, there exist pgpr-open sets G and H such that $x\in G$ but $y\notin G$ and $y\in H$ but $x\notin H$. Since every pgpr-open set is rps-open, G and H are rps-open sets such that $x\in G$ but $y\notin G$ and $y\in H$ but $x\notin H$. This shows that (X,τ) is rps-T_1.This proves (i).

The proof of (ii) follows from (i) and Proposition 4.2 of [4].

However, the converse of Proposition 4.2 is not true as shown in the following example.

Example 4.3: Let $X=\{a,b,c\}$ with $\tau=\{\varnothing,\{a\},\{b\},\{a,b\}, X\}$. It can be verified that (X,τ) is rps-T_1 but neither pre-T_1 nor pgpr-T_1.

The converse of Proposition 4.2 holds in rps-$T_{3/4}$ spaces as shown below.

Proposition 4.4: (i) If a space (X,τ) is rps-$T_{3/4}$ and rps-T_1, then it is pre-T_1.

(ii) If a space (X,τ) is rps-$T_{3/4}$ and rps-T_1, then it is pgpr-T_1.

Proof: Suppose (X,τ) is rps-$T_{3/4}$ and rps-T_1. Let $x\neq y\in X$. Since (X,τ) is rps-T_1, there exist rps-open sets G and H such that $x\in G$ but $y\notin G$ and $y\in H$ but $x\notin H$. Then $X\backslash G$ and $X\backslash H$ are rps-closed in X. Since (X,τ) is rps-$T_{3/4,}$ by Definition 2.8, $X\backslash G$ and $X\backslash H$ are pre-closed. This implies that G and H are pre-open sets such that $x\in G$ but $y\notin G$ and $y\in H$ but $x\notin H$. This shows that (X,τ) is pre-T_1. This proves (i).

The proof of (ii) follows from Proposition 4.2 of [4] and (i).

Proposition 4.5: (i) Every gpr-$T_{1/2}$ space is rps-T_1.

(iii) Every preregular-$T_{1/2}$ space is rps-T_1.

Proof: The proof of (i) follows from Lemma 4.5 of [4] and Proposition 4.2(ii).

The proof of (ii) follows from Corollary 4.6 of [4] and Proposition 4.2(ii).

The next examples show that rps-T_1 space need not be gpr-$T_{1/2}$ and preregular-$T_{1/2}$.

Example 4.6: Let X be a countably infinite set. We define the topology τ of finite complements on X by declaring open those sets with finite complements together with $\varnothing$ (and X). Then the only closed sets are $\varnothing$, X and finite sets. In particular $\{x\}$ is closed for every $x \in X$. It follows that (X,τ) is T_1 and hence it is rps-T_1. The only regular open sets are $\varnothing$ and X. This implies that every subset of X is gpr-closed. Consider any infinite set $A \neq X$. Then A is open and gpr-closed but not closed. Therefore $cl(int(A))=cl(A)=X \not\subset A$. That is A is not pre-closed. This shows that (X,τ) is not gpr-$T_{1/2}$.

Example 4.7: Let $X=\{a,b,c,d\}$ with $\tau=\{\varnothing,\{a,b\},\{a,b,c\},X\}$. It can be verified that (X,τ) is rps-T_1 but not preregular-$T_{1/2}$, since the set $\{a,b,c\}$ is gpr-closed but not rps-closed.

The following examples show that the concepts rps-T_1 and rps-$T_{1/2}$, rps-T_1 and rps-$T_{1/3}$, rps-T_1 and rps-T_b, rps-T_1 and rps-$T_{3/4}$ are independent.

Example 4.8: Let $X=\{a,b,c\}$ endowed with topology $\tau=\{\varnothing,\{a\},X\}$. Clearly (X,τ) is rps-$T_{1/2}$, rps-$T_{1/3}$, rps-T_b and rps-$T_{3/4}$ but it is not rps-T_1.

Example 4.9: Let $X=\{a,b,c,d\}$ with topology $\tau=\{\varnothing,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\},X\}$. It can be verified that (X,τ) is rps-T_1 but neither rps-$T_{1/2}$ nor rps-$T_{1/3}$.

Example 4.10: Let $X=\{a,b,c\}$ with topology $\tau=\{\varnothing,\{a,b\},X\}$. It can be verified that (X,τ) is rps-T_1 but not rps-T_b.

Example 4.11: Let $X=\{a,b,c\}$ with topology $\tau=\{\varnothing,\{a\},\{b\},\{a,b\},X\}$. It can be verified that (X,τ) is rps-T_1 but not rps-$T_{3/4}$.

Theorem 4.12: Let Y be a g-closed, open subspace of a topological space (X,τ). Then $G\cap Y$ is rps-open in Y whenever G is rps-open in X.

Proof: Let G be rps-open in X. Let $F\subseteq Y$ be rg-closed in Y such that $F\subseteq G\cap Y$. Since $F\subseteq Y\subseteq X$ and Y is g-closed and open, by using Theorem 3.4 of [9], F is rg-closed in X. Again since $F\subseteq G\cap Y\subseteq G$ and G is rps-open in X, by using Theorem 2.6, $F\subseteq spint(G)$ and so $F\subseteq Y\cap spint(G)$. Now $spint(G)=G\cap cl(int(cl(G)))$ implies that

$$\begin{aligned} Y\cap spint(G) &= Y\cap(G\cap cl(int(cl(G))))=(Y\cap G)\cap(Y\cap cl(int(cl(G)))) \\ &=(G\cap Y)\cap cl_Y(int_Y(cl_Y(G\cap Y)) \end{aligned}$$

$=spint_Y(G\cap Y)$, where int_Y, cl_Y and $spint_Y$ denote the corresponding interior, closure and semi-pre-interior in the subspace Y of X. This shows that $F\subseteq spint_Y(G\cap Y)$. By using Theorem 2.6, $G\cap Y$ is rps-open in Y.

The next theorem shows that, the g-closed open subspace of a rps-T_1 space is again a rps-T_1 space.

Theorem 4.13: Let $Y\subseteq X$ be g-closed and open in (X,τ). If (X,τ) is rps-T_1 then (Y,τ_Y) is also rps-T_1.

Proof: Let Y be g-closed and open in (X,τ). Let x and y be any two distinct points of Y. Suppose (X,τ) is a rps-T_1 space. Then there exist rps-open sets G and H such that $x\in G$ but $y\notin G$ and $y\in H$ but $x\notin H$. By using Theorem 4.12, $Y\cap G$ and $Y\cap H$ are rps-open in Y. Clearly $x\in Y\cap G$ but $y\notin Y\cap G$ and $y\in Y\cap H$ and $x\notin Y\cap H$. This proves that (Y,τ_Y) is also a rps-T_1 space.

Theorem 4.14: A topological space (X,τ) is rps-$T_{1/2}$ if and only if every subset B of X is the intersection of subsets A of X containing B such that A is semi-pre-closed or rg-open.

Proof: Let $B\subseteq X$. Let us first show that $B=\cap\{X\setminus\{x\}: x\notin B\}$. For, if $x\notin B$ then $B\subseteq X\setminus\{x\}$. That is $B\subseteq X\setminus\{x\}$ for every $x\notin B$. So we get $B\subseteq\cap\{X\setminus\{x\}: x\notin B\}$. On the other hand, let $y\in X\setminus\{x\}$ for every $x\notin B$. Suppose $y\notin B$. Then by our choice of y, we have $y\in X\setminus\{y\}$ that is impossible. This shows that $y\in B$ and $\cap\{X\setminus\{x\}: x\notin B\}\subseteq B$. That is $B=\cap\{X\setminus\{x\}: x\notin B\}$. Suppose (X,τ) is rps-$T_{1/2}$. By using Theorem 3.10 of [13], $\{x\}$ is semi-pre-open or rg-closed for every $x\in X$. That is $X\setminus\{x\}$ is rg-open or semi-pre-closed for every $x\in X$. Therefore, B is the intersection of semi-pre-closed sets or rg-open sets containing B. Conversely, suppose every subset B of X is the intersection of subsets A of X containing B such that A is semi-pre-closed or rg-open. Fix $x\in X$. Take $B=X\setminus\{x\}$. Then B is the intersection of semi-pre-closed sets or rg-open sets containing B. The only sets that contain B are B and X. Now $B=B\cap X$ and $B\neq X$. This implies that $B=X\setminus\{x\}$ is semi-pre-closed or rg-open. That is $\{x\}$ is semi-pre-open or rg-closed. By using Theorem 3.10 of [13], (X,τ) is rps-$T_{1/2}$.

Theorem 4.15: A topological space (X,τ) is rps-T_1 if and only if for every $x\in X$, $rps\text{-}cl(\{x\})=\{x\}$.

Proof: Let (X,τ) be rps-T_1 and $x\in X$. Then for each $y\neq x$, there exist rps-open sets G and H such that $x\in G$ but $y\notin G$ and $y\in H$ but $x\notin H$. Since H is rps-open, $X\backslash H$ is rps-closed and $x\in X\backslash H$ but $y\notin X\backslash H$. This implies that $y\notin$ *rps-cl*$(\{x\})$, for every $y\in X$ and $y\neq x$. Thus $\{x\}=$*rps-cl*$(\{x\})$. Conversely, suppose *rps-cl*$(\{x\})=\{x\}$ for every $x\in X$. Let x, y be any two distinct points in X. Then $x\notin\{y\}=$*rps-cl*$(\{y\})$ implies there exists a rps-closed set B_1 such that $y\in B_1$, $x\notin B_1$. This implies that $X\backslash B_1$ is a rps-open set such that $x\in X\backslash B_1$ but $y\notin X\backslash B_1$. Since $y\notin\{x\}=$*rps-cl*$(\{x\})$, there exists a rps-closed set B_2 such that $x\in B_2$, $y\notin B_2$. That is $X\backslash B_2$ is a rps-open set such that $y\in X\backslash B_2$ but $x\notin X\backslash B_2$. By Definition 4.1, (X,τ) is rps-T_1.

Theorem 4.16: Let $f:(X,\tau_1)\to(Y,\tau_2)$ be bijective.

(i) If f is rps-continuous and (Y,τ_2) is T_1, then (X,τ_1) is rps-T_1 .

(ii) If f is rps-irresolute and (Y,τ_2) is rps-T_1, then (X,τ_1) is rps-T_1 .

(iii) If f is rps-open and (X,τ_1) is rps-T_1, then (Y,τ_2) is rps-T_1 .

Proof: Let $f:(X,\tau_1)\to(Y,\tau_2)$ be bijective.

Suppose $f:(X,\tau_1)\to(Y,\tau_2)$ is rps-continuous and (Y,τ_2) is T_1. Let $x_1, x_2\in X$ with $x_1\neq x_2$. Since f is bijective, $y_1=f(x_1)\neq f(x_2)=y_2$ for some $y_1, y_2\in Y$. Since (Y,τ_2) is T_1 , choose open sets G and H such that $y_1\in G$ but $y_2\notin G$ and $y_2\in H$ but $y_1\notin H$. Since f is bijective, $x_1=f^{-1}(y_1)\in f^{-1}(G)$ but $x_2=f^{-1}(y_2)\notin f^{-1}(G)$ and $x_2=f^{-1}(y_2)\in f^{-1}(H)$ but $x_1=f^{-1}(y_1)\notin f^{-1}(H)$. Since f is rps- continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are rps-open sets in (X,τ_1). This shows that, (X,τ_1) is a rps-T_1 space. This proves (i). Suppose $f:(X,\tau_1)\to(Y,\tau_2)$ is rps-irresolute and (Y,τ_2) is a rps-T_1 space. Let x_1, $x_2\in X$ with $x_1\neq x_2$. Since f is bijective, $y_1=f(x_1)\neq f(x_2)=y_2$ for some $y_1, y_2\in Y$. Since (Y,τ_2) is a rps-T_1 space, choose rps-open sets G and H such that $y_1\in G$ but $y_2\notin G$ and $y_2\in H$ but $y_1\notin H$. Again since f is bijective, $x_1=f^{-1}(y_1)\in f^{-1}(G)$ but $x_2=f^{-1}(y_2)\notin f^{-1}(G)$ and $x_2=f^{-1}(y_2)\in f^{-1}(H)$ but $x_1=f^{-1}(y_1)\notin f^{-1}(H)$. Since f is rps-irresolute, $f^{-1}(G)$ and $f^{-1}(H)$ are rps-open sets in (X,τ_1). It follows that (X,τ_1) is rps-T_1. This proves (ii). Suppose f is rps-open and (X,τ_1) is rps-T_1. Let $y_1\neq y_2\in Y$. Since f is bijective, there exist x_1, x_2 in X such that $f(x_1)=y_1$ and $f(x_2)=y_2$ with $x_1\neq x_2$. Since (X,τ_1) is rps-T_1, there exist rps-open sets G and H in X such that $x_1\in G$ but $x_2\notin G$ and $x_2\in H$ but $x_1\notin H$. Since f is rps-open, $f(G)$ and $f(H)$ are rps-open in Y such that $y_1=f(x_1)\in f(G)$

and $y_2=f(x_2)\in f(H)$. Since f is bijective, we have $y_2=f(x_2)\notin f(G)$ and $y_1=f(x_1)\notin f(H)$. Thus (Y,τ_2) is rps-T_1. This proves (iii).

5. rps-T_2 spaces

In this section, we introduce rps-T_2 spaces and investigate their basic properties.

Definition 5.1: A topological space (X,τ) is rps-T_2 if for any two distinct points x and y of X, there exist disjoint rps-open sets G and H such that $x\in G$ and $y\in H$.

Proposition 5.2: (i) Every pgpr-T_2 space is rps-T_2.

(ii) Every pre-T_2 space is rps-T_2.

Proof: Suppose (X,τ) is pgpr-T_2. Let $x\neq y\in X$. Since (X,τ) is pgpr-T_2, by using Definition 2.5, there exist disjoint pgpr-open sets G and H such that $x\in G$ and $y\in H$. Since every pgpr-open set is rps-open, G and H are disjoint rps-open sets such that $x\in G$ and $y\in H$. This shows that (X,τ) is rps-T_2. This proves (i). The proof of (ii) follows from the fact that every pre-T_2 space is pgpr-T_2 and (i).

The converse of Proposition 4.2 holds in rps-$T_{3/4}$ spaces as shown in the following proposition.

Proposition 5.3: (i) If a space (X,τ) is rps-$T_{3/4}$ and rps-T_2, then it is pre-T_2.

(ii) If a space (X,τ) is rps-$T_{3/4}$ and rps-T_2, then it is pgpr-T_2.

Proof: Suppose (X,τ) is rps-$T_{3/4}$ and rps-T_2. Let $x\neq y\in X$. Since (X,τ) is rps-T_2, by Definition 5.1, there exist disjoint rps-open sets G and H such that $x\in G$ and $y\in H$. Then $X\setminus G$ and $X\setminus H$ are rps-closed in X. Since (X,τ) is rps-$T_{3/4}$, $X\setminus G$ and $X\setminus H$ are pre-closed. That is G and H are disjoint pre-open sets such that $x\in G$ and $y\in H$. Thus (X,τ) is pre-T_2. This proves (i). Since every pre-T_2 space is pgpr-T_2, (X,τ) is pgpr-T_2. This proves (ii).

Proposition 5.4: (i) Every gpr-$T_{1/2}$ space is rps-T_2.

(iii) Every preregular-$T_{1/2}$ space is rps-T_2.

Proof: (i) Follows from the Proposition 5.2 and Lemma 5.2 of [4].

(ii) Follows from the Proposition 5.2 and Proposition 5.6 of [4]

Theorem 5.5: Every rps-T_2 space is rps-T_1.

Proof: Let (X,τ) be a rps-T_2 space. Let $x\neq y\in X$. By Definition 5.1, there exist rps-open sets G and H such that $G\cap H=\emptyset$, $x\in G$ and $y\in H$. Since $G\cap H=\emptyset$, we have $x\notin H$ and $y\notin G$. That is $x\in G$ but $y\notin G$ and $y\in H$ but $x\notin H$. This proves that (X,τ) is rps-T_1.

However, a rps-T_1 space is not rps-T_2 as shown in the following example.

Example 5.6: Let $X=G\cup\{x_1\}\cup\{x_2\}$,where G denotes any infinite set and x_1, x_2 are two distinct points not in G. Let τ be the family of subsets of X such that (i) $A\in\tau$ if $A\subseteq G$ and (ii)$A\in\tau$ if x_1 or $x_2\in A$ but X\A contains only a finite number of G. Then τ is a topology for X. If $x,y\in X$ with $x\neq y$, then, both, any one or none of x, y may belong to G. Consequently, ({x}, {y}), ({x}, {y}$\cup$ [G\{x}]) and ({x}$\cup$G,{y}$\cup$G) are respectively, then, the pairs of pre-open sets, one containing x but not y while the other containing y but not x. Hence (X,τ) is pre-T_1. Since every pre-T_1 space is rps-T_1, (X,τ) is rps-T_1. Again let A and B be two rps-open sets such that $x_1\in A$ but $x_2\notin A$ while $x_2\in B$ but $x_1\notin B$. To prove (X,τ) is not rps-T_2, it suffices to show that $A\cap B\neq\varnothing$. Suppose A is closed. Then A is rg-closed, X\A is open and $A\subseteq A$. Since A is rps-open, by using Theorem 2.6, $A\subseteq spint(A)$. Always $spint(A)\subseteq A$. Hence $A=pint(A)$ and A is semi-pre-open. Therefore $A\subseteq cl(int(cl(A)))$. Since A is closed $A\subseteq cl(int(A))\subseteq cl(A)\subseteq A$. That is $A=cl(int(A))$. If A is not open, then $int(A)=A\setminus\{x_1\}$.

Now $cl(int(A))=cl(A\setminus\{x_1\}) = \begin{cases} A\setminus\{x_1\}\neq A \; if \; A \; is \; finite \\ A\cup\{x_1,x_2\}\neq A \; otherwise\end{cases}, \text{since } x_1\in A \; and \; x_2\notin A$

This is a contradiction to $A=cl(int(A))$. Therefore A is open. Now by the definition of τ, X\A containing x_2 contains only a finite number of members of G. This contradicts the assertion that X\A is open. So, A is not closed, and hence X\A is not open. Since $x_2\in X\setminus A$, $X\setminus(X\setminus A)=A$ cannot have only a finite number of G, by the definition of τ. That is A contains all points G except possibly finite number of points of G. Similarly, it can be shown that B contains all points of G except possibly a finite number of points of G. Hence $A\cap B\neq\varnothing$.

Theorem 5.7: A topological space (X,τ) is rps-T_2 if and only if the intersection of all rps-closed, rps-neighborhoods of each point of the space is reduced to that point.

Proof: Let (X,τ) be a rps-T_2 space and $x\in X$. Then for each $y\neq x$ in X, there exist disjoint rps-open sets U and V such that $x\in U$, $y\in V$. Now $U\cap V=\varnothing$ implies $x\in U\subseteq X\setminus V$. That is X\V is a rps-neighborhood of x. Since V is rps-open, X\V is rps-closed and rps-neighborhood of x to which y does not belong. That is there is a rps-closed, rps-neighborhood of x, which does not contain y. So we get the intersection of all rps-closed, rps-neighborhoods of x is {x}. Conversely, let x, $y\in X$ such that $x\neq y$ in X. Then by our assumption, there exists a rps-closed, rps-neighborhood V of x such that $y \notin V$. Since V is a rps-neighborhood of x, there exists a rps-open set U such that $x \in U \subseteq V$. Thus, U and X\V are disjoint rps-open sets containing x and y respectively. It follows that (X,τ) is rps-T_2.

The next theorem shows that, a g-closed open subspace of a rps-T_2 space is again a rps-T_2 space.

Theorem 5.8: Let Y be a g-closed and open subspace of X. If (X,τ) is rps-T_2 then (Y,τ_Y) is also rps-T_2.

Proof: Suppose (X,τ) is rps-T_2. Let x and y be any two distinct points in Y. Then $x,y \in X$. Since (X,τ) is rps-T_2, by using Definition 5.1, there exist disjoint rps-open sets G and H in X such that $x \in G$ and $y \in H$. By using Theorem 4.12, $Y \cap G$ and $Y \cap H$ are disjoint rps-open sets in Y. Clearly $x \in Y \cap G$ and $y \in Y \cap H$. This proves that (Y,τ_Y) is also a rps-T_2 space.

Theorem 4.8: Let $f:(X,\tau_1) \to (Y,\tau_2)$ be bijective.

(i) If f is rps-continuous and (Y,τ_2) is T_2, then (X,τ_1) is rps-T_2 .

(ii) If f is rps-irresolute and (Y,τ_2) is rps-T_2, then (X,τ_1) is rps-T_2 .

(iii) If f is rps-open and (X,τ_1) is rps-T_2, then (Y,τ_2) is rps-T_2 .

Proof: Let $f:(X,\tau_1) \to (Y,\tau_2)$ be a bijective function. Suppose (Y,τ_2) is T_2 and f is rps-continuous. Let $x_1, x_2 \in X$ with $x_1 \neq x_2$. Since f is bijective, $y_1=f(x_1) \neq f(x_2)=y_2$ for some $y_1,y_2 \in Y$. Since (Y,τ_2) is T_2, there exist disjoint open sets G and H such that $y_1 \in G$ and $y_2 \in H$. Since f is bijective, $x_1=f^{-1}(y_1) \in f^{-1}(G)$ and $x_2=f^{-1}(y_2) \in f^{-1}(H)$. Since f is rps-continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are rps-open sets in (X,τ_1). Moreover, $f^{-1}(G) \cap f^{-1}(H)=f^{-1}(G \cap H)=f^{-1}(\emptyset)=\emptyset$. Thus (X,τ_1) is rps-T_2. This proves (i). Suppose $f:(X,\tau_1) \to (Y,\tau_2)$ is rps-irresolute and (Y,τ_2) is rps-T_2. Let $x_1, x_2 \in X$ with $x_1 \neq x_2$. Since f is bijective, $y_1=f(x_1) \neq f(x_2)=y_2$ for some $y_1,y_2 \in Y$. Since (Y,τ_2) is rps-T_2, by using Definition 5.1, there exist disjoint rps-open sets G and H such that $y_1 \in G$ and $y_2 \in H$. Since f is a bijective map, we have $x_1=f^{-1}(y_1) \in f^{-1}(G)$ and $x_2=f^{-1}(y_2) \in f^{-1}(H)$. Since f is rps-irresolute, $f^{-1}(G)$ and $f^{-1}(H)$ are rps-open sets in (X,τ_1). Also $f^{-1}(G) \cap f^{-1}(H)$ $=f^{-1}(G \cap H)=f^{-1}(\emptyset)=\emptyset$.This shows that, (X,τ_1) is rps-T_2. This proves (ii). Suppose f is rps-open and (X,τ_1) is rps-T_2. Let $y_1 \neq y_2 \in Y$. Now since f is bijective, there exist x_1, x_2 in X such that $f(x_1)=y_1$, $f(x_2)=y_2$ and $x_1 \neq x_2$. Since (X,τ_1) is rps-T_2 , by Definition 5.1, there exist disjoint rps-open sets G and H in X such that $x_1 \in G$ and $x_2 \in H$. Since f is rps-open, $f(G)$ and $f(H)$ are disjoint rps-open sets in Y such that $y_1=f(x_1)\in f(G)$ and $y_2=f(x_2)\in f(H)$. Therefore, (Y,τ_2) is rps-T_2. This proves (iii).

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