Sasakian Manifold With Bi-Recurrent C- Bochner Curvature Tensor

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ABSTRACT

In the present paper we have studied Sasakian bi-recurrent manifold with vanishing C-Bochner curvature tensor. The various properties have been studied and hence utilized to derive the results with the help of Sasakian C- Bochner curvature tensor.

Key Words: Sasakian manifold, Almost contact structure.
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1. INTRODUCTION

Let M^{2m+1} be a (2m+1) dimensional differentiable manifold of class C^{∞} covered by a system of coordinate neighborhoods {U:x^h} where here and in the sequel the indices $\alpha, \beta, \gamma, \dots\dots.. h, i, j, k, \dots\dots..$ run over the range {1,2,………..2m+1}. Suppose that there are given in M^{2m+1} a tensor field F_i^h of type (1,1), a vector field f^h and a 1-form ∂_i satisfying

$$F_i^h F_j^i = -\delta_j^h + \alpha_j f^h \tag{1.1}$$

$$F_i^h f^i = 0 \tag{1.2}$$

$$\alpha_i F_j^i = 0 \tag{1.3}$$

$$\alpha_i f^i = 1 \tag{1.4}$$

such a set (F, f, α) is called an almost contact structure and M^{2m+1} endowed with an almost contact structure is called an almost contact manifold.

If we have

$$N_{ji}^h + (\partial_j \alpha_i - \partial_i \alpha_j) f^h = 0 \tag{1.5}$$

where N_{ji}^h is the Nijenhuis tensor of F_i^h defined by

$$N_{ji}^h = F_j^t \partial_t F_i^h - F_i^t \partial_t F_j^h - (\partial_j F_i^t - \partial_i F_j^t) F_t^h \tag{1.6}$$

constructed with F_k^h and $\partial_j = \partial / \partial x^j$ then the almost contact structure is said to be normal and M^{2m+1} endowed with a normal almost contact structure is called a normal almost contact manifold.

An almost contact manifold, in which a Riemannian metric g_{ij} such that

$$g_{ts} F_j^t F_i^s = g_{ji} - \partial_j \alpha_i \tag{1.7}$$

$$\alpha_i = f^j g_{ji} \tag{1.8}$$

is given, is called an almost contact metric manifold. We can easily verify that, in an almost contact metric manifold

$$F_{ji} = F_j^t g_{ti} \tag{1.9}$$

is skew symmetric.

If an almost contact metric structure satisfies

$$F_{ji} = \frac{1}{2}(\partial_j \alpha_i - \partial_i \alpha_j) \tag{1.10}$$

then the almost contact metric structure is said to be contact and a manifold M^{2m+1} endowed with a normal contact metric structure is called a Sasakian manifold.

In a Sasakian manifold, we have

$$F_i^h = \nabla_i f^h \tag{1.11}$$

$$\nabla_j F_i^h = -g_{ji} f^h + \delta_j^h f_i \tag{1.12}$$

where ∇_j denotes the operator of covariant differentiation with respect to the Christoffel symbols formed with g_{ji} and $f_i = f^j g_{ji}$

Since $F_{ji} = \nabla_j f_i$ and F_{ji} is skew symmetric, f^h is a unit killing vector field.

From equations (1.11) and (1.12) and Ricci identity we have

$$R_{kji}^h f^i = \delta_k^h f_j - \delta_j^h f_k \tag{1.13}$$

where R_{kji}^h is the curvature tensor of the manifold M^{2m+1}, from which we have

$$R_{ji} f^i = 2m f_j \tag{1.14}$$

where R_{ji} is the Ricci tensor of the manifold.

From equations (1.11), (1.12) and the Ricci identity we also find

$$R_{kjt}^h F_i^t - R_{kji}^t F_t^h = -\delta_k^h F_{ji} + \delta_j^h F_{ki} = -F_k^h g_{ji} + F_j^h g_{ki} \tag{1.15}$$

from which we have

$$R_{jt} F_i^t + R_{ljit} F^{lt} = -(2m-1) F_{ji} \tag{1.16}$$

where $F^{lt} = g^{li} F_i^t$. Using the first Bianchi identity we have from (1.16)

$$R_{ljit} F^{lt} = 2 R_{jt} F_i^t + 2(2m-1) F_{ji} \tag{1.17}$$

which shows that $R_{jt} F_i^t$ is skew symmetric and consequently we have

$$R_{jt} F_i^t = -R_{it} F_j^t \tag{1.18}$$

2. SASAKIAN MANIFOLD WITH C- BOCHNER CURVATURE TENSOR

The C- Bochner curvature tensor B_{kji}^h [1] in Sasakian manifold is given by

$$B_{kji}^h = K_{kji}^h + \frac{1}{n+3}\Big(K_{ki}\delta_j^h - K_{ji}\delta_k^h + g_{ki}K_j^h - g_{ji}K_k^h + S_{ki}f_j^h - S_{ji}f_k^h + f_{ki}S_j^h - f_{ji}S_k^h$$
$$+2S_{kj}f_i^h + 2f_{kj}S_i^h - K_{ki}\eta_j\eta^h + K_{ji}\eta_k\eta^h - \eta_k\eta_i K_j^h + \eta_j\eta_i K_k^h\Big) - \frac{\kappa+n-1}{n+3}\Big(f_{ji}f_j^h -$$
$$f_{ji}f_k^h + 2f_{kj}f_i^h\Big) - \frac{\kappa-4}{n+3}\Big(g_{ki}\delta_j^h - g_{ji}\delta_k^h\Big) + \frac{\kappa}{n+3}\Big(g_{ki}\eta_j\eta^h + \eta_k\eta_i\delta_j^h - g_{ji}\eta_k\eta^h$$
$$-\eta_j\eta_i\delta_k^h\Big) \tag{2.1}$$

Where

$$S_{kj} = f_k^h K_{hj} \tag{2.2}$$

$$S_k^i = S_{kj} g^{ji} \tag{2.3}$$

$$\kappa = \frac{K+n-1}{n+1} \tag{2.4}$$

Also we have,

$$B_{kji}^h = -B_{jki}^h, \quad B_{kjih} = B_{ihkj} \tag{2.5}$$

$B^h_{kji} + B^h_{iki} + B^h_{ikj} = 0,\ B^k_{kij} = 0$ (2.6)

$B^h_{kji}\eta_h = 0, f^s_k B^h_{sji} = f_j^{\ s} B^h_{ski}\ ,\quad f^k B^h_{kji} = 0$ (2.7)

Where

$B_{kji\,h} = B^s_{kji}\, s_{sh}\ ,\quad f^{kj} = f^j_s\, g^{sk}$ (2.8)

Tensor U^h_{kji} introduced by [2] onM^n whose component are defined by

$$U^h_{kji} = K^h_{kji} - (\rho+1)\left(g_{ji}\delta^h_k - g_{ki}\delta^h_j\right) - (g_{kj}\eta_j\eta^h + \eta_k\eta_i\delta^h_j - g_{ji}\eta_k\eta^k - \eta_j\eta_i\delta^h_k$$
$$+f_{ji}f^h_k - f_{ki}f_j^{\ h} - 2f_{kj}f_i^{\ h}) \quad (2.9)$$

Where

$\rho + 1 = \frac{\kappa}{n-1}$ (2.10)

A Sasakian manifold M^n is called locally C-Fubinian [3] when the tensor field U^h_{kji} vanishes identically on M^n. If a Sasakian manifold on M^n is locally C-Fubinian if its Ricci tensor satisfies

$K_{ji} = ag_{ji} + bb\eta_j\eta_i$ (2.11)

where

$a = \frac{K}{n-1} - 1$and$b = -\frac{K}{n-1} + n$ (2.12)

In this case manifold is said to be C-Einstein. If Sasakian manifold is locally C-Fubinian then it is C-Einstein.

From (2.1), (2.4), (2.9)and (2.10) we have

$$B^h_{kji} = U^h_{kji} + \frac{2\kappa(n+1) - 3(n+1)}{(n-1)(n+3)}\left(g_{ji}\delta^h_k - g_{ki}\delta^h_j\right) + \frac{n+\kappa+3}{n+3}(g_{kj}\eta_j\eta^h + \eta_k\eta_i\delta^h_j$$

$$-g_{ji}\eta_k\eta^k - \eta_j\eta_i\delta^h_k) + \frac{2(n+1)+\kappa}{n+3}(f_{ji}f^h_k - f_{ki}f_j^{\ h} - 2f_{kj}f_i^{\ h}) \quad (2.13)$$

Since $f^s_k K_{sj} = -f_j^{\ s}K_{sk}$ and the differential form $S = \frac{1}{2}S_{ji}\,dx^i dx^j$ are closed, therefore we can easily verify that the following equation hold good.

$S_{ji} = -S_{ij},\ S^k_{j,k} = \frac{1}{2}f_j^{\ k}K_{,k} + (K-n+1)p_j$ (2.14)

$S_{ji,k} = p_j K_{ik} - (n-1)g_{jk}p_i + f_j^{\ t}K_{ti,k}$ (2.15)

$f^t_j S_{ik,t} = -p_j S_{ki} + (n-1)f_{ij}p_k + f_j^{\ r} f_i^{\ s} K_{sk,r}$ (2.16)

$K_{ji,k} - K_{ki,j} = f_i^{\ r}S_{kj,r} - 2S_{kj}p_i + (n-1)(f_{ki}p_j - f_{ji}p_k + 2f_{kj}p_i)$ (2.17)

Where we have$K_{ji}p^i = (n-1)p_j$.

1. SASAKIAN MANIFOLD WITH BI-RECURRENT PROPERTIES

Definition (3.1) A Sasakian manifold is said to be bi-recurrent if we have

$$K^h_{kji,ab} - \lambda_{ab}K^h_{kji} = 0 \quad (3.1)$$

for some non-zero tensor λ_{ab} and is called Ricci bi-recurrent if it satisfies

$$K_{ji,ab} - \lambda_{ab}K_{ji} = 0 \quad (3.2)$$

multiplying (2.15) by g^{ij}, we have

$$K_{,ab} - \lambda_{ab}K = 0 \quad (3.3)$$

Definition (3.2) A Sasakian manifold satisfying the condition

$$B^h_{kji,ab} - \lambda_{ab} B^h_{kji} = 0 \tag{3.4}$$

for some non-zero tensor λ_{ab} will be called Sasakian manifold with bi-recurrent C- Bochner curvature tensor.

Definition (3.3) A Sasakian manifold satisfying the condition

$$U^h_{kji,ab} - \lambda_{ab} U^h_{kji} = 0 \tag{3.5}$$

for some non-zero tensor λ_{ab} will be called Sasakian manifold with bi-recurrent U^* curvature tensor.

THEOREM(3.1) Every Sasakian bi-recurrent manifold is a Sasakian manifold with bi-recurrent C- Bochner curvature tensor.
PROOF: Differentiating (2.1), covariantly with respect to x^a and then again differentiating the result thus obtain covariantly with respect to x^b, we obtain

$$B^h_{kji,ab} = K^h_{kji,ab} + \frac{1}{n+3}\big(K_{ki,ab}\delta^h_j - K_{ji,ab}\delta^h_k + g_{ki}K^h_{j,ab} - g_{ji}K^h_{k,ab} + S_{ki,ab}f^h_j - S_{ji,ab}f^h_k$$
$$+f_{ki}S^h_{j,ab} - f_{ji}S^h_{k,ab} + 2S_{kj,ab}f^h_i + 2f_{kj}S^h_{i,ab} - K_{ki,ab}\eta_j\eta^h + K_{ji,ab}\eta_k\eta^h$$
$$-\eta_k\eta_i K^h_{j,ab} + \eta_j\eta_i K^h_{k,ab}\big) - \frac{K_{,ab}}{n+3}\big(f_{ji}f^h_j - f_{ji}f^h_k + 2f_{kj}f^h_i\big) - \frac{K_{,ab}}{n+3}\big(g_{ki}\delta^h_j - g_{ji}\delta^h_k\big)$$
$$+\frac{K_{,ab}}{n+3}\big(g_{ki}\eta_j\eta^h + \eta_k\eta_i\delta^h_j - g_{ji}\eta_k\eta^h - \eta_j\eta_i\delta^h_k\big) \tag{3.6}$$

thus obtained from (3 multiplying (2.1), by λ_{ab} and subtracting the result.6), we have

$$B^h_{kji,ab} - \lambda_{ab}B^h_{kji} = K^h_{kji,ab} - \lambda_{ab}K^h_{kji} + \frac{1}{n+3}\big[(K_{ki,ab} - \lambda_{ab}K_{ki})\delta^h_j - (K_{ji,ab} - \lambda_{ab}K_{ji})\delta^h_k$$
$$+g_{ki}(K^h_{j,ab} - \lambda_{ab}K^h_j) - g_{ji}(K^h_{k,ab} - \lambda_{ab}K^h_k) + (S_{ki,ab} - \lambda_{ab}S_{ki})f^h_j$$
$$-(S_{ji,ab} - \lambda_{ab}S_{ji})f^h_k + f_{ki}(S^h_{j,ab} - \lambda_{ab}S^h_j) - f_{ji}(S^h_{k,ab} - \lambda_{ab}S^h_k)$$
$$+2(S_{kj,ab} - \lambda_{ab}S_{kj})f^h_i + 2f_{kj}(S^h_{i,ab} - \lambda_{ab}S^h_i) - (K_{ki,ab} - \lambda_{ab}K_{ki})\eta_j\eta^h$$
$$+(K_{ji,ab} - \lambda_{ab}K_{ji})\eta_k\eta^h - \eta_k\eta_i(K^h_{j,ab} - \lambda_{ab}K^h_j) + \eta_j\eta_i(K^h_{k,ab} - \lambda_{ab}K^h_k)\big]$$
$$-\frac{(K_{,ab} - \lambda_{ab}\kappa)}{n+3}\big[\big(f_{ji}f^h_j - f_{ji}f^h_k + 2f_{kj}f^h_i\big) - \big(g_{ki}\delta^h_j - g_{ji}\delta^h_k\big)$$
$$+\big(g_{ki}\eta_j\eta^h + \eta_k\eta_i\delta^h_j - g_{ji}\eta_k\eta^h - \eta_j\eta_i\delta^h_k\big] \tag{3.7}$$

If the manifold is bi-recurrent then from (2.2), (2.3), (3.1), (3.2), and (3.3) the equation (3.7) reduces to

$$B^h_{kji,ab} - \lambda_{ab}B^h_{kji} = 0$$

which shows that the manifold will also a Sasakian manifold with bi-recurrent C-Bochner Curvature tensor.

THEOREM (3.2) The necessary and sufficient condition that a Sasakian manifold is Sasakian Ricci bi-recurrent is that

$$B^h_{kji,ab} - \lambda_{ab} B^h_{kji} = K^h_{kji,ab} - \lambda_{ab} K^h_{kji}$$

PROOF: Let the manifold be Sasakian Ricci bi-recurrent then the relation (3.2), is satisfied. From the equation (3.7) in view of equation (2.2), (2.3), (3.2) and (3.3), gives

$$B^h_{kji,ab} - \lambda_{ab} B^h_{kji} = K^h_{kji,ab} - \lambda_{ab} K^h_{kji} \quad - \quad (3.8)$$

Conversely, if in the Sasakian manifold (3.8), is satisfied then we have from (3.7)

$$\{[(K_{ki,ab} - \lambda_{ab} K_{ki})\delta^h_j - (K_{ji,ab} - \lambda_{ab} K_{ji})\delta^h_k + g_{ki}(K^h_{j,ab} - \lambda_{ab} K^h_j) - g_{ji}(K^h_{k,ab} - \lambda_{ab} K^h_k)$$

$$+(S_{ki,ab} - \lambda_{ab} S_{ki})f^h_j - (S_{ji,ab} - \lambda_{ab} S_{ji})f^h_k + f_{ki}(S^h_{j,ab} - \lambda_{ab} S^h_j) - f_{ji}(S^h_{k,ab} - \lambda_{ab} S^h_k)$$

$$+2(S_{kj,ab} - \lambda_{ab} S_{kj})f^h_i + 2f_{kj}(S^h_{i,ab} - \lambda_{ab} S^h_i) - (K_{ki,ab} - \lambda_{ab} K_{ki})\eta_j \eta^h$$

$$+(K_{ji,ab} - \lambda_{ab} K_{ji})\eta_k \eta^h - \eta_k \eta_i (K^h_{j,ab} - \lambda_{ab} K^h_j) + \eta_j \eta_i (K^h_{k,ab} - \lambda_{ab} K^h_k)]$$

$$-(K_{,ab} - \lambda_{ab} K)[(f_{ji} f^h_j - f_{ji} f^h_k + 2f_{kj} f^h_i) - (g_{ki}\delta^h_j - g_{ji}\delta^h_k) + (g_{ki}\eta_j \eta^h + \eta_k \eta_i \delta^h_j - g_{ji}\eta_k \eta^h - \eta_j \eta_i \delta^h_k)]\} = 0$$

which yields with the help of (2.2), and (2.3)

$$K^h_{kji,ab} - \lambda_{ab} K^h_{kji} = 0$$

i.e the manifold is Sasakian Ricci bi-recurrent.

THEOREM (3.3) If a Sasakian manifold satisfies any two of the following properties
(i). The manifold is Sasakian Ricci bi-recurrent
(ii). The manifold is Sasakian manifold with bi-recurrent Bochner Curvature tensor
(iii). The manifold is U^* bi-recurrent; then it must also satisfy the third.

Proof: Differentiating (2.9), covariantly with respect to x^a and then again differentiating the result thus obtain covariantly with respect to x^b, we obtain

$$B^h_{kji,ab} = U^h_{kji,ab} + \frac{K_{,ab}}{(n+3)}\left[\frac{1}{(n-1)}(g_{ji}\delta^h_k - g_{ki}\delta^h_j) + (g_{kj}\eta_j \eta^h + \eta_k \eta_i \delta^h_j\right.$$

$$\left. -g_{ji}\eta_k \eta^k - \eta_j \eta_i \delta^h_k) + (f_{ji} f^h_k - f_{ki} f^h_j - 2f_{kj} f^h_i)\right] \quad (3.9)$$

Trasvecting (2.9) with λ_{ab} and subtracting the result thus obtained from (3.9), we have

$$B^h_{kji,ab} - \lambda_{ab} B^h_{kji} = U^h_{kji,ab} - \lambda_{ab} U^h_{kji} + \frac{K_{,ab} - \lambda_{ab} K}{(n+3)}\left[\frac{1}{(n-1)}(g_{ji}\delta^h_k - g_{ki}\delta^h_j)\right.$$

$$\left. +(g_{kj}\eta_j \eta^h + \eta_k \eta_i \delta^h_j - g_{ji}\eta_k \eta^k - \eta_j \eta_i \delta^h_k) + (f_{ji} f^h_k - f_{ki} f^h_j - 2f_{kj} f^h_i)\right] \quad (3.10)$$

The statement of the above theorems follows in view of the equations (2.2),(2.3),(2.7), (3.1),(3.2),(3.4),(3.5) and (3.10).

Theorem (3.4): In a Sasakian bi-recurrent manifold with vanishing C-Bochner curvature tensor we have

$$\lambda_{ka} K_{ji} - \lambda_{ja} K_{kl} = \eta_k [\eta_j K_{ai} - \eta_i K_{aj}] - \eta_j [\eta_k K_{ai} - \eta_i K_{ak}] - 2\eta_i [\eta_k K_{aj} - \eta_j K_{ak}]$$

$$+ \frac{K_{,h}}{2(n+1)}\{\eta_k [\phi_{ja}\delta^h_i - \phi_{ia}\delta^h_j + 2\phi_{ji}\delta^h_a + (g_{ia} - \eta_i \eta_a)\phi^h_j$$

$$-(g_{ja} - \eta_j \eta_a)\phi^h_i] - \eta_j [\phi_{ka}\delta^h_i - \phi_{ia}\delta^h_k + 2\phi_{ki}\delta^h_a$$

$$+(g_{ia} - \eta_i \eta_a)\phi^h_k - (g_{ka} - \eta_k \eta_a)\phi^h_i] - 2\eta_i [\phi_{ka}\delta^h_j - \phi_{ja}\delta^h_i$$

$$+2\phi_{kj}\delta_a^h+\left(g_{ja}-\eta_j\eta_a\right)\phi_k^h-\left(g_{ka}-\eta_k\eta_a\right)\phi_j^h]$$

$$-\lambda_{ha}K[(g_{ki}-\eta_k\eta_i)\delta_j^h-\left(g_{ji}-\eta_j\eta_i\right)\delta_k^h+\phi_{ki}\phi_j^h-\phi_{ji}\phi_k^h+2\phi_{kj}\phi_i^h]\} \quad (3.11)$$

Proof: Differentiating (2.1) covariantly and using (2.14),(2.15),(2.16) and (2.17), we have

$$(n+3)\nabla_h B_{kji}^h=(n+2)\left(\nabla_k K_{ji}-\nabla_j K_{ki}\right)-\phi_k^r\phi_j^s(\nabla_r K_{si}-\nabla_s K_{ri})+2\phi_i^s\phi_k^r\nabla_s K_{rj}$$
$$+\eta^r\left(\eta_k\nabla_r K_{ki}-\eta_j\nabla_r K_{ki}\right)-(n+2)\eta_k S_{ji}+n\eta_j S_{ki}+2(n+1)\eta_i S_{kj}$$
$$+\frac{1}{n+1}\left(g_{ki}\eta_j-g_{ji}\eta_k\right)\eta^r\nabla_r K+\frac{n-1}{2(n+1)}\{(g_{ki}-\eta_k\eta_i)\nabla_j K$$
$$-\left(g_{ji}-\eta_j\eta_i\right)\nabla_k K+(\phi_{ki}\phi_j^r-\phi_{ji}\phi_k^r+2\phi_{kj}\phi_i^r)\nabla_r K\}$$
$$+(n+1)\{(n+2)\eta_k\phi_{ji}-n\eta_j\phi_{ki}-2(n+1)\eta_i\phi_{kj}\} \quad (3.12)$$

Transvecting (3.12) with $\phi_l^k\phi_m^j$ and adding the resultant equation to [3.12] we have

$$B_{lmi,h}^h+\phi_l^k\phi_m^j B_{kji,h}^h=\left(K_{mi,l}-K_{li,m}\right)-\phi_l^k\phi_m^j\left(K_{ji,k}-K_{kl,j}\right)+(n-1)(\eta_l\phi_{mi}-\eta_m\phi_{li})$$
$$-\eta_l S_{mi}+\eta_m S_{li}+\frac{1}{2(n+3)}(g_{li}\eta_m-g_{mi}\eta_l)\eta^h K_{,h} \quad (3.13)$$

also we have

$$\phi_l^k\phi_m^j B_{kji}^h=-B_{mli,h}^h\ , \quad (3.14)$$

from which we have

$$K_{ji,k}-K_{kl,j}-\phi_k^r\phi_j^s\left(K_{si,r}-K_{rl,s}\right)-\eta_k S_{ji}+\eta_j S_{kl}+\frac{1}{2(n+3)}(g_{ki}\eta_j-g_{ji}\eta_k)\eta^r K_{,r}$$
$$+(n-1)\left(\eta_k\phi_{ji}-\eta_j\phi_{kl}\right)=0 \quad (3.15)$$

Contracting the above equation with η^k and $\eta^k g^{ji}$, we find respectively

$$\eta^h K_{,h}=0\ ,\quad \eta^h K_{ji,h}=0 \quad (3.16)$$

From which

$$\frac{(n+3)}{(n-1)}B_{kji,h}^h=K_{ji,k}-K_{kl,j}-\eta_k\{S_{ji}-(n-1)\phi_{ji}\}+\eta_j\{S_{ki}-(n-1)\phi_{kl}\}$$
$$+2\eta_i\{S_{kj}-(n-1)\phi_{kj}\}+\frac{1}{2(n+1)}\{(g_{ki}-\eta_k\eta_i)\delta_j^h-\left(g_{ji}-\eta_j\eta_i\right)\delta_k^h$$
$$+\phi_{ki}\phi_j^h-\phi_{ji}\phi_k^h+2\phi_{kj}\phi_i^h\}K_{,h} \quad (3.17)$$

Thus, in a Sasakian manifold with vanishing C-Bochner curvature tensor, we get

$$K_{ji,k}-K_{kl,j}=\eta_k\{S_{ji}-(n-1)\phi_{ji}\}-\eta_j\{S_{ki}-(n-1)\phi_{kl}\}-2\eta_i\{S_{kj}-(n-1)\phi_{kj}\}$$
$$-\frac{1}{2(n+1)}\{(g_{ki}-\eta_k\eta_i)\delta_j^h-\left(g_{ji}-\eta_j\eta_i\right)\delta_k^h+\phi_{ki}\phi_j^h-\phi_{ji}\phi_k^h+$$
$$2\phi_{kj}\phi_i^h\}K_{,h} \quad (3.18)$$

$$S_{ji,k}=\eta_j K_{ki}-\eta_i K_{kj}+\frac{1}{2(n+1)}\{\phi_{jk}\delta_i^h-\phi_{ik}\delta_j^h+2\phi_{ji}\delta_k^h+(g_{ik}-\eta_i\eta_k)\phi_j^h$$
$$-\left(g_{jk}-\eta_j\eta_k\right)\phi_i^h\}K_{,h} \quad (3.19)$$

Differentiating (3.19)covariantly with respect to a

$$K_{ji,ka}-K_{kl,ja}=\eta_k S_{ji,a}-\eta_j S_{ki,a}-2\eta_i S_{kj,a}$$
$$-\frac{1}{2(n+1)}\{(g_{ki}-\eta_k\eta_i)\delta_j^h-\left(g_{ji}-\eta_j\eta_i\right)\delta_k^h+\phi_{ki}\phi_j^h-\phi_{ji}\phi_k^h+$$
$$2\phi_{kj}\phi_i^h\}K_{,ha} \quad (3.20)$$

Above equation in view of (3.19) reduces to(3.11).

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